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# ABSTRACTS

**Roman Badora**
University of Silesia, Katowice, Poland

## On approximately multiplicative functions

We discuss a form of a function $f$ between a semigroup $S$ and a topological algebra $\mathcal{A}$ for which the following difference

$$S \times S \ni (x, y) \longmapsto f(x + y) - f(x)f(y) \in \mathcal{A}$$

is bounded in cases when the algebra $\mathcal{A}$ has an orthogonal basis and when the Banach algebra $\mathcal{A}$ as a normed space is a reflexive space.

**Anna Bahyrycz**
Pedagogical University, Kraków, Poland

## On stability and hyperstability of an equation characterizing multi-additive mappings

Given two semigroups $G$ and $H$, we say that a function $f\colon G^n \to H$ is *multi-additive* if it is additive (satisfies Cauchy's functional equation) in each variable. K. Ciepliński in [1] has reduced the system of $n$ equations defining the multi-additive mapping to a single functional equation and has shown the Hyers-Ulam stability both of this system and this equation using the direct method.

In the talk, we present some results on the Hyers-Ulam stability and hyperstability of this equation (characterizing multi-additive mappings). To prove our main result we apllied the fixed point approach.

### Reference

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## Karol Baron
University of Silesia, Katowice, Poland
(joint work with **Rafał Kapica** and **Janusz Morawiec**)

### On Lipschitzian solutions to an inhomogeneous linear iterative equation

Basing on iteration of random-valued functions we study the problems of the existence, uniqueness and continuous dependence of Lipschitzian solutions $\varphi$ of the equation

$$\varphi(x) = F(x) - \int_\Omega \varphi(f(x,\omega))P(d\omega),$$

where $P$ is a probability measure on a $\sigma$-algebra of subsets of $\Omega$.

## Mihály Bessenyei
University of Debrecen, Debrecen, Hungary
(joint work with **Anna Bella Popovics**)

### Convexity without convex combinations

Separation theorems play a central role in the theory of Functional Inequalities. The importance of Convex Geometry has led to the study of convexity structures induced by Beckenbach families. The aim of the present talk is to replace recent investigations into the context of an axiomatic setting, for which Beckenbach structures serve as models. Besides the alternative approach, some new results (whose classical correspondences are well-known in Convex Geometry) are also presented.

## Zoltán Boros
University of Debrecen, Debrecen, Hungary
(joint work with **Zsolt Páles**)

### Characterizations of Wright-convexity with respect to a real subfield

Let $K$ denote a subfield of $\mathbb{R}$, $X$ be a non-trivial linear space over $K$, and $D$ be a non-empty, $K$-convex and $K$-algebraically open subset of $X$. We call $f\colon D \to \mathbb{R}$ $K$-convex, $K$-Wright-convex, respectively, if, for every $x, y \in D$ and $\lambda \in [0,1] \cap K$, $f$ satisfies the inequality

$$f(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda)f(y),$$
$$f(\lambda x + (1-\lambda)y) + f((1-\lambda)x + \lambda y) \le f(x) + f(y),$$

respectively. The radial $\mathbb{Q}$-derivative

$$d_{\mathbb{Q}} f(x,u) = \lim_{\mathbb{Q}_+ \ni r \to 0} \frac{f(x+ru) - f(x)}{r} \in \mathbb{R}$$

of a Jensen-convex function $f\colon D \to \mathbb{R}$ exists for every $u \in X$ and $x \in D$. We prove that the following assertions are equivalent for $f\colon D \to \mathbb{R}$.

(i) $f$ is $K$-Wright-convex.

(ii) There exists a $K$-convex function $g\colon D \to \mathbb{R}$ and an additive mapping $A\colon X \to \mathbb{R}$ such that $f(x) = g(x) + A(x)$ for all $x \in D$.

(iii) $f$ is Jensen-convex and the mapping $r \mapsto d_{\mathbb{Q}} f(x+ru, u)$ ($r \in K$ such that $x + ru \in D$) is increasing for all $x \in D$ and $u \in X$.

## Janusz Brzdęk

Pedagogical University, Kraków, Poland
(joint work with **El-Sayed El-Hady**)


### On some functional equations in the queuing theory

Several issues are discussed that are connected to the problem of finding solutions of some functional equations of the form

$$C_1(x,y)P(x,y) = C_2(x,y)P(x,0) + C_3(x,y)P(0,y) + C_4(x,y)P(0,0) + C_5(x,y),$$

where $C_i$, $i = 1,\dots,5$, are given functions in two complex variables $x, y$ and the unknown function $P$ is a probability generating function defined for every $x, y$ belonging to the closed unit disc. Such functional equations arise in particular in the queuing theory.


## Pál Burai

University of Debrecen, Debrecen, Hungary
(joint work with **Attila Házy** and **Tibor Juhász**)


### A composite functional equation from algebraic aspect

In this talk we discuss the composite functional equation

$$f(x + 2f(y)) = f(x) + y + f(y),$$

where $f\colon G \to G$ and $G$ is an Abelian group.


## Jacek Chmieliński

Pedagogical University, Kraków, Poland
(joint work with **Paweł Wójcik**)


### Approximate preservation of orthogonality by two mappings

Let $f$ and $g$ be unknown functions between two inner product spaces. The stability of the orthogonality equation

$$\langle f(x)|g(y)\rangle = \langle x|y\rangle$$

as well as the approximate orthogonality preserving property

$$x \perp y \implies fx \perp^{\varepsilon} gy$$

will be considered.


## Jacek Chudziak

University of Rzeszów, Rzeszów, Poland


### On composite equations related to conjugation of some families of transformations

We consider the solutions of the functional equation

$$f(g(x)y + h(x)) = G(x)f(y) + H(x). \tag{1}$$

Equation (1), arising naturally in the studies on invariant utility functions, is related to conjugation of some families of transformations. The following generalization of (1)

$$f(\phi_1(x)\phi_2(y) + \phi_3(x)) = \Psi_1(x)\Psi_2(y) + \Psi_3(x),$$

will be considered as well.


## Zoltán Daróczy

University of Debrecen, Debreacen, Hungary


### On a problem related to a general functional equation

Some special cases of the next problem related to a general functional equation is studied. Let $X$ and $Y$ be nonempty sets and let $\circ$ and $*$ be binary operations on $X$ such that $x \circ y \neq x * y$ holds whenever $x \neq y$ and $x, y \in X$. Assume that the unknown function $f\colon X \to Y$ satisfies the next property:

$$\textit{If } f(x) \neq f(y), \textit{ then } f(x \circ y) = f(x * y). \tag{P}$$

The question is, what can we state about functions $f$ fulfilling property (P).


## EL-Sayed El-Hady

Innsbruck University, Innsbruck, Austria
Suez Canal University, Ismailia, Egypt
(joint work with **Wolfgang Förg-Rob** and **Janusz Brzdęk**)


### On a functional equation arising from inventory control of database systems

Recently, a certain structure of two-place functional equations popped up from many interesting applications like e.g. wireless networks. The general structure of this particular class of equations is given by

$$f(x, y) = \frac{c_2(x, y)f(x, 0) + c_3(x, y)f(0, y) + c_4(x, y)f(0, 0) + c_5(x, y)}{c_1(x, y)},$$

where the functions $c_i(x, y)$ for $i = 1, \dots, 5$ are given polynomials in two complex variables $x$ and $y$. The unknown functions $f(x, y)$, $f(x, 0)$ and $f(0, y)$ are generating functions of some sequences of interest in many applications see [1], [2] e.g.. There is no general solution theory available to solve such general class of equations. In this talk I will introduce a solution of a special case of this general form. Such a functional equation arises from a queueing model which has applications in inventory control of database systems. The solution is obtained by assuming full symmetry on the system parameters, by using the theory of boundary value problems see [3] e.g., and by using conformal mapping.

#### References


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[3] J.W. Cohen, O.J. Boxma, *Boundary value problems in queueing system analysis*, North-Holland Mathematics Studies, 79. North-Holland Publishing Co., Amsterdam, 1983.

## Włodzimierz Fechner

University of Silesia, Katowice, Poland


# Regularity of solutions of Hlawka's functional inequality

Let $(X, +)$ be a topological Abelian group. We discuss regularity of solutions $f\colon X \to \mathbb{R}$ of the Hlawka's functional inequality

$$f(x+y) + f(y+z) + f(x+z) \leq f(x+y+z) + f(x) + f(y) + f(z),$$

postulated for all $x, y, z \in X$. In particular, we provide conditions which imply the continuity of $f$. We continue and develop our studies from [1].

### Reference


[1] W. Fechner, *Hlawka's functional inequality*, Aequationes Math. **87** (2014), no. 1-2, 71–87.



## Gian Luigi Forti

Università degli Studi di Milano, Milano, Italy


# A comparison among methods for proving stability

The aim of this talk is to present and compare the various methods used in literature for proving Ulam–Hyers stability of functional equations: the direct method in its various forms, the shadowing method, the fixed point approach and the use of invariant means.

### References


[1] L. Cădariu, V. Radu, *Fixed points and the stability of Jensen's functional equation*, JIPAM. J. Inequal. Pure Appl. Math. **4** (2003), no. 1, Article 4, 7pp. (electronic).

[2] L. Cădariu, V. Radu, *On the stability of the Cauchy functional equation: a fixed point approach*, Iteration theory (ECIT '02), 43–52, Grazer Math. Ber. **346**, Karl-Franzens-Univ. Graz, Graz, 2004.

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## Harald Fripertinger
Karl-Franzens-Universität, Graz, Austria

### A remark on $n$-associative power series

Last year the general solution of the equation of $n$-associativity

$$F(F(x_1,\ldots,x_n),x_{n+1},\ldots,x_{2n-1}) = F(x_1,F(x_2,\ldots,x_{n+1}),x_{n+2},\ldots,x_{2n-1}) = \ldots$$
$$= F(x_1,\ldots,x_{n-1},F(x_n,\ldots,x_{2n-1}))$$

for $F(x_1,\ldots,x_n) \in \mathbb{C}[\![x_1,\ldots,x_n]\!]$, $n \in \mathbb{N}$, $n \geq 3$, with $F(0,\ldots,0) = 0$ was presented in my talk.

If $F(x_1,\ldots,x_n) = x_1 + x_n + \ldots$, then we proved that there exist $\rho \in \mathbb{C}$, an $n-1$-th root of unity of order $n-1$, and a formal power series $f(x) = x + \ldots \in \mathbb{C}[\![x]\!]$, so that

$$F(x_1,\ldots,x_n) = f^{-1}\big(f(x_1) + \rho f(x_2) + \rho^2 f(x_3) + \ldots + \rho^{n-2} f(x_{n-1}) + f(x_n)\big).$$

Generalizing a result of [1], it is possible to determine the power series $f$ by differentiation which is much more elegant than the previous proof.

#### Reference

[1] H. Fripertinger, J. Schwaiger, *On one-dimensional formal group laws in characteristic zero*, Aequationes Math. (2014) DOI: 10.1007/s00010-014-0282-6.

## Roman Ger
University of Silesia, Katowice, Poland

### Convexity and a modified difference property

It is known (see [2]) that the class of convex functions fails to have the difference property in the sense of N.G. de Bruijn [1] even in the real case. It seems that the requirement upon the function $f\colon \mathbb{R} \to \mathbb{R}$ in question to satisfy the condition

$$\Delta_h f \quad \text{is convex for all} \quad h \in \mathbb{R},$$

is too strong (and hence inadequate) in this case. Therefore, trying to get the assertion desired, we shall confine ourselves to positive $h$'s only.

#### References

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## Attila Gilányi
University of Debrecen, Debrecen, Hungary

### Computer assisted methods for functional equations
### Dedicated to the 90$^{\text{th}}$ birthday of János Aczél

One of the first problems related to the solution of functional equations with computers was formulated by János Aczél. Starting with the solution of this problem, we consider computer assisted methods for investigations of functional equations. We discuss some general features of these methods and we point out their connection to the recently introduced concept of mathability.

## Dorota Głazowska

University of Zielona Góra, Zielona Góra, Poland
(joint work with **Janusz Matkowski**)


### Subcommuting and commuting real homographic functions


If the difference of two real homographic functions is nonnegative, then it is constant. Motivated by this property we determine all pairs of subcommuting (supercommuting) real homographic functions. We also show that simple modification of subcommuting (supercommuting) functions transforms them into commuting ones. Moreover we deal with one parameter families of comparable commuting homographic functions. In particular we show that a generalized iteration group of comparable homographic functions coincides with the family of translations of the identity function.


**References**


[1] E.F. Beckenbach, *Generalized convex functions*, Bull. Amer. Math. Soc. **43** (1937), no. 6, 363–371.

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[3] Zs. Páles, *21. Problem in Report of The Forty-second International Symposium on Functional Equations*, Aequationes Math. **69** (2005), 192.



## Moshe Goldberg

Technion - Israel Institute of Technology, Haifa, Israel


### Subnorms on Cayley-Dickson algebras


We begin this talk by a brief account of the Cayley-Dickson algebras and by recalling that the *radius* on these algebras is given by the Euclidean norm. With this observation we turn to two related topics: a variant of the Gelfand formula, and stability of continuous subnorms.



## László Horváth

University of Pannonia, Veszprém, Hungary


### Infinite refinements of the discrete Jensen's inequality defined by recursion


In this talk we give refinements of the discrete Jensen's inequality for convex and mid-convex functions defined by recursion. Conditions are given for strict inequality which is rare in this topic. In some cases explicit formulas are obtained. The results contain and generalize earlier statements. As an application we define some new quasi-arithmetic means and study their (strict) monotonicity.



## Wojciech Jabłoński

Rzeszow University of Technology, Rzeszów, Poland


### Steinhaus-type property for a boundary of a slice


The notion of an open slice appears, among others, in characterizations of denting points of bounded closed and convex sets in a Banach space. We show that if $\mathcal{S}_1(x_0, x_0^*, \delta)$ is a boundary

of a slice in a real normed space $X$, i.e. if

$$\mathcal{S}_1(x_0, x_0^*, \delta) := \{x \in S_1 : \ x_0^*(x) > 1 - \delta\}$$

for suitably $x_0^* \in S_1^* \subset X^*$, then $\mathcal{S}_1(x_0, x_0^*, \delta)$ has the so called *Stainhaus-type property*, i.e.

$$\operatorname{int}(\mathcal{S}_1(x_0, x_0^*, \delta) + \mathcal{S}_1(x_0, x_0^*, \delta)) \neq \emptyset.$$

**Justyna Jarczyk**
University of Zielona Góra, Zielona Góra, Poland
(joint work with **Zoltán Daróczy** and **Witold Jarczyk**)

## On marginal joints of means

Given two means on adjacent intervals the problem is to define a mean on the union of these intervals, which is a common extension of both the means. We propose the notion of marginal joint providing a pretty wide family of such extended means.

**Witold Jarczyk**
University of Zielona Góra, Zielona Góra, Poland
(joint work with **Zsolt Páles**)

## Convexity in an abelian semigroup setting

We propose two parallel notions of convexity of functions in the abelian semigroup setting. In the talk characterizations of both the notions and results comparising them are presented.

**Gergely Kiss**
Budapest University of Technology, Budapest, Hungary
(joint work with **Vincze Csaba**)

## Inhomogeneous linear functional equation related to Sosztok's problem

We investigate the solution of the following functional equation

$$F(y) - F(x) = (y - x) \sum_{i=1}^{n} a_i f(\alpha_i x + \beta_i y), \tag{1}$$

where $F, f$ are unknown functions. Szostok presented some papers about solutions of integral approximations such like (1). He showed that if the parameters where chosen like $\alpha_i = 1 - \beta_i$ then the additive solutions are of the form $c \cdot x$, and asked when we can guarantee this conclusion in the general case? We consider the solutions of (1) for arbitrary $\alpha_i, \beta_i \in \mathbb{C}$. We will give a structural description of the solutions, especially focusing on the additive solutions of (1). For this we generalize the theory of homogeneous linear functional equations which uses discrete spectral sythesis and analysis.

## Tomasz Kochanek

University of Warsaw, Warszawa, Poland

### Approximately order zero maps between C*-algebras

We deal with the problem whether an approximately zero-product preserving (order zero) map between C*-algebras must be close to a zero-product preserving map. More precisely, for given C*-algebras $A$, $B$ and a symmetric, bounded, linear operator $\phi\colon A \to B$ satisfying $\|\phi(x)\phi(y)\| \le \varepsilon\|x\|\|y\|$ for all self-adjoint $x, y \in A$ with $xy = 0$, we ask whether there exists a zero-product preserving operator $\psi\colon A \to B$ with $\|\phi - \psi\| \le \delta(\varepsilon)$, where $\delta(\varepsilon) \to 0$ as $\varepsilon \to 0$. We show this holds true in the case where $A$ is nuclear (amenable as a Banach algebra) and $B$ is isomorphically a dual Banach $B$-bimodule (or a two-sided ideal of such). We also show that such a property is valid in some cases when $A = \mathcal{B}(\ell_2)$, but is not valid in general.

We shall also briefly discuss possible consequences of our results in the theory of nuclear dimension of C*-algebras (a notion introduced by W. Winter and J. Zacharias in 2010) and cohomology of Banach algebras (in the spirit of B.E. Johnson's theory of amenability).

## Zygfryd Kominek

University of Silesia, Katowice, Poland

(joint work with **Justyna Sikorska**)

### Alienation of the logarithmic and exponential functional equations

The aim of the talk is to present a general solution of the functional equation

$$f(xy) - f(x) - f(y) = g(x + y) - g(x)g(y),$$

which is strictly connected with the problem of alienation of logarithmic and exponential Cauchy's functional equations for real functions of a real variable. We solve this equation both in case where we consider all real variables, and - taking into account the nature of a logarithmic function - for non-zero variables. In the latter case the problem turns out to be much more complicated. Generally, we do not assume any regularity conditions on $f$ and $g$. But if $g(1) = 1$ and $f(1) = 0$, unfortunately, the method of the proof which we use forces us to assume the continuity of $g$ at the origin.

## Károly Lajkó

University of Debrecen, Debrecen, Hungary

(joint work with **Tamás Glavosits**)

### Pexiderizations of some logarithmic functional equations

The main aim of this talk is to give the general solution of the functional equation

$$\gamma(y(x + 1)) + g(x(y + 1)) = h(x) + h(y) \qquad (x, y \in T_+)$$

for functions $\gamma, g, h\colon T_+ \to A$, where $T_+$ is the set of positive elements in an ordered field $T$ and $A$ is a uniquely 2-divisible abelian group.

As a direct consequence of our main result it is obtained e.g. that every solution $\gamma\colon T_+ \to A$ of the equation

$$\gamma(y(x + 1)) + \gamma(x(y + 1)) = \gamma(x(x + 1)) + \gamma(y(y + 1)) \qquad (x, y \in T_+)$$

with condition $\gamma(1) = 0$ satisfies the Cauchy logarithmic equation

$$\gamma(xy) = \gamma(x) + \gamma(y) \qquad (x, y \in T_+).$$

## Hugo Leiva

University of Andes, Mérida, Venezuela
(joint work with **Nelson Merentes**)


### Controllability of the semilinear heat equation with impulses and delays on the state


For many control systems in real life, impulses and delays are intrinsic properties that do not modify their controllability. So we conjecture that under certain conditions the abrupt changes and delays as perturbations of a system do not modify certain properties such as controllability. In this regard, in this paper we prove the interior approximate controllability of the Semilinear Heat Equation with Impulses and Delays.


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## Zbigniew Leśniak

Pedagogical University, Kraków, Poland


### On the topological conjugacy of flows of generalized Reeb homeomorphisms


We study the problem of topological conjugacy in a class of flows of Brouwer homeomorphisms. We give a necessary and sufficient condition for flows of generalized Reeb homeomorphisms to be topologically conjugated. The condition describes the relationship between the transition maps of the flows. Moreover, we consider an invariant which distinguishes flows conjugate to the standard generalized Reeb flow.



## László Losonczi

University of Debrecen, Debrecen, Hungary


### Equality of Páles means


Let $f, g\colon I \to \mathbb{R}$ be continuous functions such that $g$ is positive and $f/g$ is strictly monotone on the nonempty open interval $I$, and let $\mu$ be a probability measure on the Borel subsets of $[0,1]$. The two variable mean $M_{f,g;\mu}\colon I^2 \to I$, called Páles mean is defined by

$$M_{f,g;\mu}(x,y) := \left(\frac{f}{g}\right)^{-1}\left(\frac{\int_0^1 f(tx+(1-t)y)\,d\mu(t)}{\int_0^1 g(tx+(1-t)y)\,d\mu(t)}\right) \qquad (x,y \in I).$$

In the talk we discuss the equality problem

$$M_{f,g;\mu}(x,y) = M_{k,l;\nu}(x,y) \qquad (x,y \in I).$$

of such means.

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[2] L. Losonczi, *Equality of two variable Cauchy mean values*, Aequationes Math. **65** (2003), no. 1-2, 61–81.

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[4] L. Losonczi, Zs. Páles, *Equality of two-variable functional means generated by different measures*, Aequationes Math. **81** (2011), no. 1-2, 31–53.

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## Radosław Łukasik

University of Silesia, Katowice, Poland

(joint work with **Paweł Wójcik**)


### Orthogonality equation with two unknown functions

Let $\mathcal{H}$, $\mathcal{K}$ be Hilbert spaces over the same field $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$. Let $\langle\cdot|\cdot\rangle$ denotes the inner product. We will solve the generalized orthogonality equation

$$\forall_{x,y\in\mathcal{H}} \ \langle f(x)|g(y)\rangle = \langle x|y\rangle \,, \tag{1}$$

with two unknown functions $f, g\colon \mathcal{H} \to \mathcal{K}$.


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## Judit Makó

University of Miskolc, Miskolc-Egyetemváros, Hungary

(joint work with **Pál Burai** and **Attila Házy**)


### On Wright convexity type inequalities

Let $D$ be a convex nonempty subset of a linear space $X$. We say that a symmetric function $f\colon D \times D \to \mathbb{R}$ is generalized Wright convex, if

$$f(tx + (1-t)y, (1-t)x + ty) \leq f(x, y) \qquad (x, y \in D,\ t \in [0, 1]).$$

If the above inequality stands only one $t \in ]0, 1[$, we say that $f$ is generalized $t$-Wright convex. In this talk, we give some basic properties of generalized Wright convex functions, we prove a Bernstein–Doetsch type theorem, moreover we will look for connections between generalized Wright convex functions and Hermite–Hadamard type inequalities.

## Gyula Maksa

University of Debrecen, Debrecen, Hungary

(joint work with **Zsolt Páles**)


### Associativity and the Hosszú equation

Inspired by the observation that the continuous, cancellative, and associative operations $(x, y) \mapsto x + y - xy$ and $(x, y) \mapsto xy$ on $]0, 1[\times]0, 1[$ in the Hosszú equation

$$f(x + y - xy) + f(xy) = f(x) + f(y) \qquad (x, y \in ]0, 1[)$$

have the additional property that their pointwise sum is the ordinary addition, we investigate the functional equation

$$\varphi^{-1}(\varphi(x) + \varphi(y)) + \psi^{-1}(\psi(x) + \psi(y)) = x + y \qquad (x, y \in I) \tag{1}$$

where $I \subseteq \mathbb{R}$ (the reals) is an interval of positive length, and $\varphi, \psi \colon I \to \mathbb{R}$ are strictly monotonic and continuous functions. We present all such a solutions of (1) and the general solution $f \colon I \to \mathbb{R}$ of the Hosszú type equation

$$f\big(\varphi^{-1}(\varphi(x) + \varphi(y))\big) + f\big(\psi^{-1}(\psi(x) + \psi(y))\big) = f(x) + f(y) \qquad (x, y \in I)$$

where $\varphi, \psi \colon I \to \mathbb{R}$ are strictly monotonic and continuous functions satisfying (1).


## Jean-Luc Marichal

University of Luxembourg, Luxembourg, Luxembourg

(joint work with **Bruno Teheux**)


### Generalizations and variants of associativity for variadic functions: a survey

For any nonempty set $X$, we let $X^*$ denote the set of all tuples on $X$. We endow $X^*$ with the concatenation operation for which we use the juxtaposition notation. For every string $\mathbf{x}$ and every integer $n \geqslant 1$, the power $\mathbf{x}^n$ stands for the string obtained by concatenating $n$ copies of $\mathbf{x}$. The length of a string $\mathbf{x}$ is denoted by $|\mathbf{x}|$.

Recall that a function $F \colon X^* \to X^*$ is said to be

- *associative* if, for every $\mathbf{x}, \mathbf{y}, \mathbf{z} \in X^*$, we have $F(\mathbf{xyz}) = F(\mathbf{x}F(\mathbf{y})\mathbf{z})$;
- *barycentrically associative* if, for every $\mathbf{x}, \mathbf{y}, \mathbf{z} \in X^*$, we have the equality $F(\mathbf{xyz}) = F(\mathbf{x}F(\mathbf{y})^{|\mathbf{y}|}\mathbf{z})$.

More generally, for any nonempty set $Y$, a function $F \colon X^* \to Y$ is said to be

- *preassociative* if, for every $\mathbf{x}, \mathbf{y}, \mathbf{y}', \mathbf{z} \in X^*$, we have $F(\mathbf{xyz}) = F(\mathbf{xy}'\mathbf{z})$ whenever $F(\mathbf{y}) = F(\mathbf{y}')$.
- *barycentrically preassociative* if, for every $\mathbf{x}, \mathbf{y}, \mathbf{y}', \mathbf{z} \in X^*$ such that $|\mathbf{y}| = |\mathbf{y}'|$, we have $F(\mathbf{xyz}) = F(\mathbf{xy}'\mathbf{z})$ whenever $F(\mathbf{y}) = F(\mathbf{y}')$.

In this presentation we survey the most recent results obtained on the properties above.

**References**

[1] E. Lehtonen, J.-L. Marichal, B. Teheux, *Associative string functions*, Asian-Eur. J. Math. **7** (2014), no. 4, 1450059, 18pp.

[2] J.-L. Marichal, B. Teheux, *Barycentrically associative and preassociative functions*, Acta Math. Hungar. **145** (2015), no. 2, 468–488.

## Janusz Matkowski

University of Zielona Góra, Zielona Góra, Poland

### Commutativity of integral quasi-arithmetic means on probability spaces

Let $f$ and $g$ be real-valued continuous strictly monotonic functions defined on a real interval $I$, and let $(X, \mathcal{L}, \lambda)$ and $(Y, \mathcal{M}, \mu)$ be probability spaces.

We say that the pair $(f, g)$ is a $(\lambda, \mu)$-*switcher* if, for every $\mathcal{L} \times \mathcal{M}$-measurable function $h \colon X \times Y \to I$,

$$f^{-1}\Big(\int_X f\Big(g^{-1}\Big(\int_Y g \circ h \, d\mu\Big)\Big) \, d\lambda\Big) = g^{-1}\Big(\int_Y g\Big(f^{-1}\Big(\int_Y f \circ h \, d\lambda\Big)\Big) \, d\mu\Big),$$

(that is if the integral quasi-arithmetic means of the generators $f$ and $g$ are permutable).

Under some general conditions, one can prove, that $(f, g)$ is a $(\lambda, \mu)$-switcher if, and only if, $g = af + b$ for some $a, b \in \mathbb{R}$, $a \neq 0$.

**Reference**

[1] P. Leonetti, J. Matkowski, S. Tringali, *On the commutation of generalized means on probability spaces*, submitted.

## Jolanta Misiewicz

Warsaw University of Technology, Warszawa, Poland

### Generalized convolutions and the Levi-Civita functional equation

A commutative and associative $\mathcal{P}$-valued binary operation $\diamond$ defined on $\mathcal{P}_+^2$ is called a *generalized convolution* if for all $\lambda, \lambda_1, \lambda_2 \in \mathcal{P}_+$ and $a \geqslant 0$ we have:

(i) $\delta_0 \diamond \lambda = \lambda$;

(ii) $(p\lambda_1 + (1-p)\lambda_2) \diamond \lambda = p(\lambda_1 \diamond \lambda) + (1-p)(\lambda_2 \diamond \lambda)$ whenever $p \in [0, 1]$;

(iii) $T_a(\lambda_1 \diamond \lambda_2) = (T_a\lambda_1) \diamond (T_a\lambda_2)$;

(iv) if $\lambda_n \to \lambda$ then $\lambda_n \diamond \eta \to \lambda \diamond \eta$ for all $\eta \in \mathcal{P}$ and $\lambda_n \in \mathcal{P}_+$,

(v) there exists a sequence $(c_n)_{n\in\mathbb{N}}$ of positive numbers such that the sequence $T_{c_n}\delta_1^{\diamond n}$ converges to a measure different from $\delta_0$,

where $\to$ denotes weak convergence of probability measures.

We want to characterize such general convolutions for which the convolution of two one-point measures $\delta_x, \delta_1$ is a convex linear combination of $n$ fixed measures and only coefficients of linear combination depend on $x$. More exactly: there exists $\lambda_0, \ldots, \lambda_n$ such that for all $x \in [0, 1]$

$$\delta_x \diamond \delta_1 = \sum_{k=0}^{n} p_k(x)\lambda_k,$$

for some functions $p_k\colon [0,1] \to [0,1]$ such that $p_0(x) + \ldots + p_n(x) = 1$ for all $x \in [0,1]$. In the language of generalized characteristic function this lieds to the following

$$\varphi(xt)\varphi(t) = \sum_{k=1}^{n} p_k(x)\Phi_k(t), \quad x \in [0,1],\ t \geqslant 0.$$

We show that this equation can be written in the form of multiplicative Levi-Civita functional equation and then we characterize for each $n \geqslant 2$ the set of generalized convolutions with the considered property.


## Janusz Morawiec

University of Silesia, Katowice, Poland


### Wavelets and functional equations

The talk will be divided into three parts. In the first one we will introduce the concept of wavelet bases. Next, we will discuss how functional equations (called refinement equations) are involved in the construction of wavelet bases. Finally, we will present various areas of mathematics in which refinement equations play an important role.


## Kazimierz Nikodem

University of Bielsko-Biala, Bielsko-Biała, Poland
(joint work with **Milica Klaričić Bakula**)


### The converse Jensen inequality for strongly convex functions

A function $f\colon D \to \mathbb{R}$ is called *strongly convex* with modulus $c$ if

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) - ct(1-t)(x-y)^2$$

for all $x, y \in I$ and $t \in [0,1]$. The following counterpart of the converse Jensen inequality is proved:

Theorem
*If $f\colon I \to \mathbb{R}$ is strongly convex with modulus $c$, then*

$$\begin{aligned} f(\bar{x}) &\leq \sum_{i=1}^{n} t_i f(x_i) - c\sum_{i=1}^{n} t_i (x_i - \bar{x})^2 \\ &\leq \frac{M-\bar{x}}{M-m} f(m) + \frac{\bar{x}-m}{M-m} f(M) - c(M-\bar{x})(\bar{x}-m) \end{aligned}$$

*for all $x_1, \ldots, x_n \in [m, M] \subset I$, $t_1, \ldots, t_n > 0$ with $t_1 + \ldots + t_n = 1$ and $\bar{x} = t_1 x_1 + \ldots + t_n x_n$.*

**Andrzej Olbryś**
University of Silesia, Katowice, Poland

## On separation theorem for delta-subadditive and delta-superadditive mappings

Let $(X, \cdot)$ be a semigroup, and let $(Y, \|\cdot\|)$ be a real Banach space. Motivated by the dissertation of L. Veselý and L. Zajiček [3] R. Ger in [2] considered the following functional inequality

$$\|F(x) + F(y) - F(x \cdot y)\| \leq f(x) + f(y) - f(x \cdot y), \quad x, y \in X.$$

If a pair $(F, f)$ satisfies the above inequality, then we say that a map $F\colon X \to Y$ is delta-subadditive with a control function $f\colon X \to \mathbb{R}$. If a pair $(-G, -g)$ satisfies the above inequality, then we say that $G$ is delta-superadditive with a control function $g$. Inspired by methods contained in [1] we generalize the well known separation theorem for subadditive and superadditive functionals to the case of delta-subadditive and delta-superadditive mappings. We also consider the problem of supporting delta-subadditive maps by additive ones. As a consequence of these theorems we obtain the stability result for Cauchy's equation.

### References

[1] Z. Gajda, Z. Kominek, *On separations theorems for subadditive and superadditive functionals*, Studia Math. **100** (1991), no. 1, 25–38.

[2] R. Ger, *On functional inequalities stemming from stability questions*, General Inequalities 6, ISNM, **103** Birkhäuser Verlag, Basel, 1992, 227–240.

[3] L. Veselý, L. Zajiček, *Delta-convex mappings between Banach spaces and applications*, Dissertationes Math. (Rozprawy Mat.) **289** (1989), 52pp.

**Jolanta Olko**
Pedagogical University, Kraków, Poland

## On a system of functional inclusions

Inspired by the papers concerning a pair of functional inequalities characterizing polynomials, we consider multifunctions satisfying two simultaneous conditional functional inclusions of the form

$$\begin{cases} F(x+a) \subset F(x) + \sum_{j=0}^{k} \alpha_j x^j, \\ F(x+b) \subset F(x) + \sum_{j=0}^{k} \beta_j x^j, \end{cases}$$

which is also related to the notion of microperiodic function. Under some additional assumptions, an explicit formula for the solution to the above system of inclusions is given. As an application, we obtain a counterpart of the result in the single-valued case.

**Zsolt Páles**
University of Debrecen, Debrecen, Hungary

## Characterization of differences of $\omega$-convex functions

In the talk we introduce a notion of a second-order variation which enables us to characterize functions that are differences of $\omega$-convex functions, where $\omega = (\omega_1, \omega_2)$ is a two-dimensional Chebyshev system. The analogous question for higher-order convexity is also considered and a conjecture is formulated.

## Paweł Pasteczka

University of Warsaw, Warszawa, Poland

### Iterated quasi-arithmetic mean-type mappings

For a family of quasi-arithmetic means satisfying certain smoothness condition we majorize the speed of convergence of the iterative sequence of self-mappings having a mean on each entry, described in the definition of Gaussian product, to relevant mean-type mapping. We apply this result to approximate any continuous function which is invariant with respect to such a self-mappings.

## Magdalena Piszczek

Pedagogical University, Kraków, Poland

(joint work with **Janusz Brzdęk**)

### Fixed points of set-valued mappings

We present a fixed point theorem for nonlinear operators, acting on some function spaces (of set-valued maps), which satisfy suitable inclusions. We also show that it can be easily applied in proving that near the set-valued mappings satisfying various functional inclusions there exist single valued solutions of the corresponding functional equations.

## Wolfgang Prager

Karl-Franzens-Universität, Graz, Austria

(joint work with **Jens Schwaiger**)

### Banach limit solutions of the inhomogeneous Cauchy equation

Given suitable $\varphi\colon \mathbb{R}\times\mathbb{R}\to\mathbb{R}$, this talk deals with solutions of

$$f(x+y)-f(x)-f(y)=\varphi(x,y) \tag{1}$$

(and of several equations being related to (1)), which are Banach limits of certain sequences. Thereby some results presented in [1] are generalized.

**Reference**

[1] W. Prager, L. Reich, *Solutions of the inhomogeneous Cauchy equation*, Results Math. **54** (2009), no. 1-2, 149–165.

## Teresa Rajba

University of Bielsko-Biala, Bielsko-Biała, Poland

### Higher order convex ordering properties and inequalities of the Hermite-Hadamard type

We give new necessary and sufficient conditions for higher order convex ordering [4]. This result generalizes the Ohlin Lemma (1969) [3] and the Levin-Stečkin theorem (1960) [2] on convex ordering, as well as the Denuit-Lefèvre-Shaked theorems (1998) [1] on higher order convex ordering. The obtained result can be useful in the study of the Hermite-Hadamard type inequalities and in particular inequalities between the quadrature operators.

**References**

[1] M. Denuit, C. Lefèvre, M. Shaked, *The s-convex orders among real random variables, with applications*, Math. Inequal. Appl. **1** (1998), no. 4, 585–613.

[2] V.I. Levin, S.B. Stečkin, *Inequalities*, Amer. Math. Soc. Transl. (2) **14** (1960), 1–29.

[3] J. Ohlin, *On a class of measures of dispersion with application to optimal reinsurance*, ASTIN Bulletin **5** (1969), 249–266.

[4] T. Rajba, *Higher order convex ordering properties and inequalities of the Hermite-Hadamard type*, Preprint ATH, No 27, April 2015.



## Daniel Reem

University of São Paulo, São Carlos, Brazil


### Remarks on the Cauchy functional equation and variations of it

The talk will examine various aspects related to the Cauchy functional equation $f(x+y) = f(x) + f(y)$. In particular, it will consider its solvability and its stability relative to subsets of multi-dimensional Euclidean spaces and tori. Several new types of regularity conditions are introduced, such as a one in which a complex exponent of the unknown function is locally measurable. An initial value approach to treating this equation is considered too. The analysis is extended to related equations such as the Jensen equation, the multiplicative Cauchy equation, and the Pexider equation.


## Ludwig Reich

Karl-Franzens-Universität, Graz, Austria


### Some remarks on the group of Möbius transformations

Motivated by J. Matkowski's talk at the European Conference on Iteration Theory 2014 we study maximal abelian subgroups, iteration groups, reversibility and involutions in the Möbius group. Our main tool are conjugations and normal forms with respect to conjugation.


## Maciej Sablik

University of Silesia, Katowice, Poland


### Aggregating means

We are going to speak on aggregating means, that is on using the generalized bisymmetry equation to get results on the form of means in some function spaces. We present a general approach, and get the shape of bisymmetrical, continuous and increasing mean in spaces of integrable functions and bounded functions.


**References**

[1] J. Aczél, *Lectures on Functional Equations and their Applications*, Academic Press, New York - London, 1966.

[2] Gy. Maksa, *Solution of generalized bisymmetry type equations without surjectivity assumptions*, Aequationes Math. **57** (1999), no. 1, 50–74.

[3] A. Münnich, Gy. Maksa, R.J. Mokken, *n-variable bisection*, J. Math. Psych. **44** (2000), no. 4, 569–581.

## Jens Schwaiger

Karl-Franzens-Universität, Graz, Austria
(joint work with **Wolfgang Prager**)


### Stability of functional equations for polynomials

The solution of the equation $p(x)+\sum_{i=1}^{n}a_ip(x+\rho_iy)+\sum_{j=1}^{l}b_jp(\sigma_jy)=0$ are generalized polynomials of degree at most $n$. The general solution heavily depends on the parameters $a_i,\rho_i,b_j,\sigma_j$. Here the stability of this equation is investigated, i.e. for given suitable $\varphi$ the inequality $\|f(x)+\sum_{i=1}^{n}a_if(x+\rho_iy)+\sum_{j=1}^{l}b_jf(\sigma_jy)\|\le\varphi(x,y)$ is considered. The method seems to be not standard: At first it is shown that $f$ is "close" to some generalized polynomial $p$ of degree at most $n$; and then it is shown that that $p$ is a solution of the equation above.


## Ekaterina Shulman

University of Silesia, Katowice, Poland


### On Levi-Civita equations in distributions

We will discuss the Levi-Civita equation

$$f(x+y)=\sum_{k=1}^{n}u_k(x)v_k(y)$$

and some more general addition theorems in the class of distributions (generalized functions). As a consequence, the description of classical solutions of such equations on domains of $\mathbb{R}^n$ will be obtained.


## Jaroslav Smítal

Silesian University, Opava, Czech Republic
(joint work with **Ludwig Reich** and **Marta Štefánková**)


### Generalized Dhombres equation and periodic points

We consider continuous solutions $f\colon\mathbb{R}_+\to\mathbb{R}_+=(0,\infty)$ of the functional equation $f(xf(x))=\varphi(f(x))$, where $\varphi$ is a given continuous map from $\mathbb{R}_+$ to $\mathbb{R}_+$. After more than fifteen years of research we are able to give a complete solution of the following problem: what are possible periods of periodic points contained in the range $R_f$ of a solution? The answer essentially depends on the type of solution $f$: it is *singular* if there are $a$ and $b$ such that $0<a\le b<\infty$, $f|_{(0,a)}>1$, $f|_{[a,b]}\equiv1$, and $f|_{(b,\infty)}<1$; all other solutions are *regular*.

The main results can be found in the references listed below.

#### References


[1] L. Reich, J. Smítal, M. Štefánková, *The converse problem of the generalized Dhombres functional equation*, Math. Bohem. **130** (2005), 301–308.

[2] L. Reich, J. Smítal, *Functional equation of Dhombres type – a simple equation with many open problems*, J. Difference Equ. Appl. **15** (2009), no. 11-12, 1179–1191.

[3] L. Reich, J. Smítal, M. Štefánková, *Functional equation of Dhombres type in the real case*, Publ. Math. Debrecen **78** (2011), no. 3-4, 659–673.

[4] L. Reich, J. Smítal, M. Štefánková, *Singular solutions of the generalized Dhombres functional equation*, Results Math. **65** (2014), no. 1-2, 251–261.

[5] J. Smítal, M. Štefánková, *On regular solutions of the generalized Dhombres equation*, Aequationes Math. **89** (2015), no. 1, 57–61.

[6] L. Reich, J. Smítal, M. Štefánková, *On regular solutions of the generalized Dhombres equation II*, Results Math. **67** (2015), no. 3-4, 521–528.



## Peter Stadler

University of Innsbruck, Innsbruck, Austria

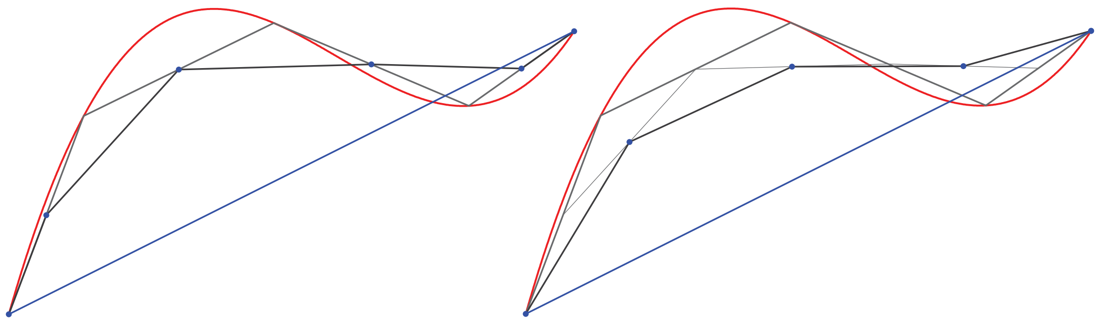

### Curve shortening by short rulers

We look at homomorphisms $h\colon (\mathbb{R},+) \to (G,\circ)$ on a Lie group $G$:

$$h(s+t) = h(s)\circ h(t), \quad h(0) = e, \quad h(1) = g.$$

The restriction of $h$ to the interval $[0,1]$ is a geodesic.

On Riemannian manifolds geodesics are locally shortest lines. The problem is to construct long geodesics. But any curve connecting starting point and end point can be shortened by using a ruler which allows to construct short geodesics:



In normed vector spaces, the curve converges to the straight line if it's shortened iterative. This result can be generalized to some Riemannian manifolds.


## Marta Štefánková

Silesian University, Opava, Czech Republic

(joint work with **Jana Dvořáková** and **Natascha Neumärker**)


### On $\omega$-limit sets of non-autonomous dynamical systems with a uniform limit of type $2^\infty$

The talk is devoted to the study of properties of $\omega$-limit sets of non-autonomous dynamical systems on compact metric spaces given by sequences of maps which uniformly converge to a continuous map $f$.

We show that, for systems defined on compact metric spaces, if an $\omega$-limit set $\tilde{\omega}$ of the non-autonomous system is a subset of the set $P(f)$ of periodic points of $f$ then $\tilde{\omega}$ is necessarily the union of finitely many disjoint connected sets which are cyclically mapped to one another. Using this result we answer a question posed by Cánovas in [1] by proving that, if an interval map $f$ has only finite $\omega$-limit sets, then the $\omega$-limit set $\tilde{\omega}$ of the non-autonomous system is a subset of the set of periodic points of $f$. We also show that a similar result applies to systems on trees but not on graphs with loops.

#### Reference


[1] J.S. Cánovas, *On $\omega$-limit sets of non-autonomous discrete systems*, J. Difference Equ. Appl. **12** (2006), no. 1, 95–100.

## László Székelyhidi
University of Debrecen, Debrecen, Hungary
(joint work with **Jose Maria Almira**)

### On the graph of additive and related functions

By classical results in the theory of functional equations the graphs of noncontinuous additive functions behave pathologically. In this talk we present some recent results concerning this problem for additive and related functions.

## Tomasz Szostok
University of Silesia, Katowice, Poland

### Inequalities connected with numerical differentiation

Write the well known Hermite-Hadamard inequality

$$f\Big(\frac{x+y}{2}\Big) \leq \frac{1}{y-x}\int_x^y f(t)\,dt \leq \frac{f(x)+f(y)}{2}$$

in the form

$$f\Big(\frac{x+y}{2}\Big) \leq \frac{F(y)-F(x)}{y-x} \leq \frac{f(x)+f(y)}{2} \tag{1}$$

with $F' = f$.

In [1] the middle term from (1) was replaced by more general formulas used in numerical differentiation. Thus inequalities involving expressions of the form

$$\frac{\sum_{i=1}^n a_i F(\alpha_i x + \beta_i y)}{y-x},$$

where $\sum_{i=1}^n a_i = 0$, $\alpha_i + \beta_i = 1$, $i = 1, \ldots, n$ and $F' = f$ were considered. In the current talk we obtain inequalities for expressions of the form

$$\frac{\sum_{i=1}^n a_i F(\alpha_i x + \beta_i y)}{(y-x)^2}$$

which are used to approximate the second order derivative of $F$.

**Reference**

[1] A. Olbryś, T. Szostok, *Inequalities of the Hermite-Hadamard type involving numerical differentiation formulas*, Results Math. **67** (2015), 403–416.

## Bruno Teheux
University of Luxembourg, Luxembourg, Luxembourg
(joint work with **Jean-Luc Marichal**)

### Strongly barycentrically associative and preassociative functions

Let $X$ be a nonempty set and $X^*$ be the free monoid generated by $X$. Recall that a function $F\colon X^* \to X \cup \{\varepsilon\}$ is barycentrically associative if the function value of a string does not change when replacing every letter of a substring of consecutive letters with the value of this substring.

In this talk, we investigate the weaker property of *strong barycentric associativity* which stipulates that the function value of a string does not change when replacing every letter of *any* substring with the value of this substring. Equivalently, a function $F\colon X^* \to X \cup \{\varepsilon\}$ is strongly barycentrically associative if and only if it satisfies the equation

$$F(\mathbf{xyz}) = F\big(F(\mathbf{xz})^{|\mathbf{x}|}\mathbf{y}F(\mathbf{xz})^{|\mathbf{z}|}\big), \qquad \mathbf{xyz} \in X^*.$$

We also investigate a variant of strong barycentric ssociativity called *strong barycentric preassociativity* which does not involve composition of functions in its definition. We establish links between strong barycentric associativity and strong barycentric preassociativity. We recall a variant of Kolmogoroff-Nagumo's characterization of the class of quasi-arithmetic means based on the strong barycentric associativity property, and we generalize this characterization to strongly barycentrically-preassociative functions.


## Peter Volkmann

Karlsruher Institut für Technologie, Karlsruhe, Germany


### Tabor groupoids and stability

Let $S$ be a groupoid. For $x \in S$ the powers $x^{2^n}$ $(n \in \mathbb{N} = \{1, 2, 3, \ldots\})$ are recursively defined by $x^{2^1} = x^2 = x \cdot x$, $x^{2^{n+1}} = x^{2^n} \cdot x^{2^n}$. $S$ is called a *Tabor groupoid*, if for $x, y \in S$ there always exists $n \in \mathbb{N}$ such that $(x \cdot y)^{2^n} = x^{2^n} \cdot y^{2^n}$. – We present some results on Tabor groupoids and we discuss their application to Pólya-Szegő-Hyers-Ulam-stability of functional equations.


## Szymon Wąsowicz

University of Bielsko-Biala, Bielsko-Biała, Poland


### Convex multifunctions and local affine selections

It is known that a convex multifunction defined on a real interval with compact and convex values on the real line admits the affine selection. In general, for a convex multifunction defined on a convex subset of a vector space with (necessarily convex) values in another vector space, this statement need not be true. Nevertheless, if a domain is a convex subset of $\mathbb{R}^n$, then any convex multifunction admits a local affine selection. We consider a problem of whether it is true in the general setting.


## Marek Cezary Zdun

Pedagogical University, Kraków, Poland

(joint work with **Dorota Krassowska**)


### Embeddability of homeomorphisms of the circle in set-valued iteration groups

We propose here a new approach to the problem of embeddability. For a given homeomorphism $F\colon \mathbb{S}^1 \to \mathbb{S}^1$ without periodic points we construct some substitute of an iteration group, namely the unique special set-valued iteration group $\{F^t\colon \mathbb{S}^1 \to \mathrm{cc}[\mathbb{S}^1],\ t \in \mathbb{R}\}$, which is regular in a sense and in which $F$ can be embedded, i.e. $F(x) \in F^1(x)$. This set-valued group is a single-valued if and only if $F$ is a minimal homeomorphism. We also determine a maximal countable and dense subgroup $T \subset \mathbb{R}$ such that $\{F^t \colon \mathbb{S}^1 \to \mathrm{cc}[\mathbb{S}^1],\ t \in T\}$ has continuous selections $\{f^t \colon \mathbb{S}^1 \to \mathbb{S}^1,\ t \in T\}$ being the embeddings of $F$, that is iteration groups satisfying the condition $f^1 = F$. If there exists a nonmeasurable embedding $\{f^t \colon \mathbb{S}^1 \to \mathbb{S}^1,\ t \in \mathbb{R}\}$ of $F$, then there exists an additive function $\gamma \colon \mathbb{R} \to T$ modulo 1 such that $f^t(z) \in F^{\gamma(t)}(z)$, $t \in \mathbb{R}$.